Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

STAT 50 MIDTERM #1

FALL 2019

PLEASE TURN OFF YOUR CELL PHONE AND DO NOT HAVE IT ON YOUR DESK

CALCULATORS ALLOWED FOR ARITHMETIC ONLY

SHOW ALL WORK AND CLEARLY INDICATE YOUR ANSWERS ON THIS TEST!!!

1. The amount of warpage in a type of wafer used in the manufacture of integrated circuits has a mean of 1.3 mm and a standard deviation of 0.1 mm. A random sample (SRS) of 200 wafers is drawn. Let be the sample mean. Find and .

2. A factory production line is manufacturing bolts using three machines, A, B, and C. Of the total output, machine A is responsible for 25%, machine B for 35%, and machine C for the rest. It is known from previous experience with the machines that 5% of the output from machine A is defective, 4% from machine B, and 2% from machine C. Round your final answers to three decimal places.

1. What is the probability that a randomly chosen bolt will be defective?
2. A bolt is chosen at random from the production line and found to be defective. What is the probability that it came from machine C?

3. Two resistors, with resistances and , which are random variables, are connected in series, so the total resistance is . is has a mean of 100 and a standard deviation of , and hash a mean of and a standard deviation of . Assuming the resistors operate independently:

1. What is the mean of ?
2. What is standard deviation
3. If a technician accidently wrote down , would it change the mean and/or standard deviation? Calculate the mean and standard deviation in this scenario.

4. You have 100 emails in your inbox, 60 are spam, and 40 are not. Of the 60 spam emails, 35 contain the word “free”. Of the rest, 3 contain the word “free”.

1. Fill in the table below with the appropriate counts.

|  |  |  |  |
| --- | --- | --- | --- |
|  | “free” | no “free” | total |
| Spam |  |  |  |
| not spam |  |  |  |
| Total |  |  |  |

b.(5 Points) Given that an email contains the word “free”, determine the probability that it is spam.

5. A computer sends a packet of information along a channel and waits for a return signal acknowledging that the packet has been received. If no acknowledgement is received within a certain time, the packet is re-sent. Let the random variable X represent the number of times the packet is sent. Assume the probability mass function of X is given by

p(x) = 

1. Find the value of the constant c so that p(x) is a properly-defined probability mass function.

Hint: It is not required, but it may help to make a table which displays the pmf of X, that is, a table showing the possible values of X in one row and the associated probability in another row.

1. Find p(2) = P(X = 2).
2. Find the mean number of times the packet is sent.
3. Find the variance of the number of times the packet is sent.
4. Sketch the graph of the cumulative distribution function (cdf), F(x). Be sure to label your axes and show the scale on each axis.

6. Cylindrical cans are manufactured according to specifications regarding their height and radius. Let R be the radius of a randomly sampled can in mm. The probability mass function of R is given by P(R=30) = 0.6 and the P(R = 31) = 0.4. Based on the above, E(R) = 30.4 and V(R) = 0.24. The circumference of a can is given by C = 2πR. Find the expected value and variance of the circumference. Leave your answers in terms of .

7. The distribution of a random variable X is given in the table below.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| X | 1 | 4 | 6 | 8 | 10 | 14 |
| P(X = x) | 0.020 | 0.222 | 0.165 |  | .317 |  |

1. If P(X = 8) = 3P(X = 14), find P(X = 8) and P(X = 14).
2. Find the Expected Value (Mean) of X.
3. Find the Variance of X.

8. Computer chips often contain surface imperfections. For a certain type of computer chip, the probability mass function of the number of defects is presented in the following table.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| X | 0 | 1 | 2 | 3 | 4 |
| P(X=x) | 0.4 | 0.3 | 0.15 | 0.10 | 0.05 |

1. Find
2. Find
3. Find

9. The error in the length of a part (absolute value of the difference between the actual length and the target length), in mm, is a random variable with probability density function

1. What is the probability that the error is less than .5 mm?
2. Find the variance of the error.
3. Find the cumulative distribution function of the error.

10. Computer keyboard failures can be attributed to electrical defects or mechanical defects. A repair facility currently has 25 failed keyboards, 6 of which have electrical defects and 19 of which have mechanical defects.

1. How many ways are there to randomly select 5 of these keyboards for a thorough inspection (without regard to order)?
2. In how many ways can a sample of 5 keyboards be selected so that exactly two have an electrical defect?
3. If a sample of 5 keyboards is randomly selected, what is the probability that at least 4 of these will have a mechanical defect?

11. The manager of an IT department needs to hire 3 technicians. There are 12 candidates who are all equally qualified. The 12 candidates consists of 7 men and 5 women.

1. What is the probability that the three who are hired consists of either all men or all women? You do not need to compute a decimal answer.
2. What is the probability that the three who are hired consists of two women and one man? You do not need to compute a decimal answer.

12. A gas station earns $2.60 in revenue for each gallon of regular gas it sells, $2.75 for each gallon of midgrade gas, and $2.90 for each gallon of premium gas. Let denote the number of gallons of regular, midgrade, and premium gasoline sold in a day. Assume , respectively.

1. (5 Points) Find the mean daily revenue.
2. (5 Points) Assuming are independent, find the standard deviation of the daily revenue.

13. Machines A and B produce 10% and 90%, respectively of the production of acomponent intended for the motor industry. From experience, it is known that the probability that machine A produces a defective component is 0.01 while the probability that machine B produces a defective component is 0.05. If a component is selected at random from a day’s production and is found to be defective, use Bayes’ theorem to find the probability that it is made by machine B.

14. Elongation (in percent) of steel plates treated with aluminum is a random variable with probability density function given by:

1. Find the mean elongation.
2. Find the variance of the elongation.
3. Find the cumulative distribution function (cdf) of . That is, find .
4. Find .